2.1 Definition range

Since, as is well known, the divisor of a division must not be zero, it is necessary to check for which x-values the denominator or divisor of a fractional rational function is zero.

To do this, set the denominator = 0 and resolve to x.

The found numbers represent the definition gaps.

Example:

$$f(x) = \frac{x^2 - 3x + 4}{x - 2}$$

Set the denominator = 0 and resolve to x.

x - 2 = 0 → x = 2

2 is therefore the definition gap, since the function is not defined at this point.

This results in the following definition range: ID = IR \ {2}

2.2. Points of discontinuity

By definition, a pole is a definition gap, near which the function values go towards infinity.

We examine the definition gaps in more detail to determine whether a pole or a gap exists.

If we insert the definition gap in the denominator of the function, we get the value 0. If we insert the definition gap in the numerator, we get either 0 or a value not equal to 0.

It is now valid:

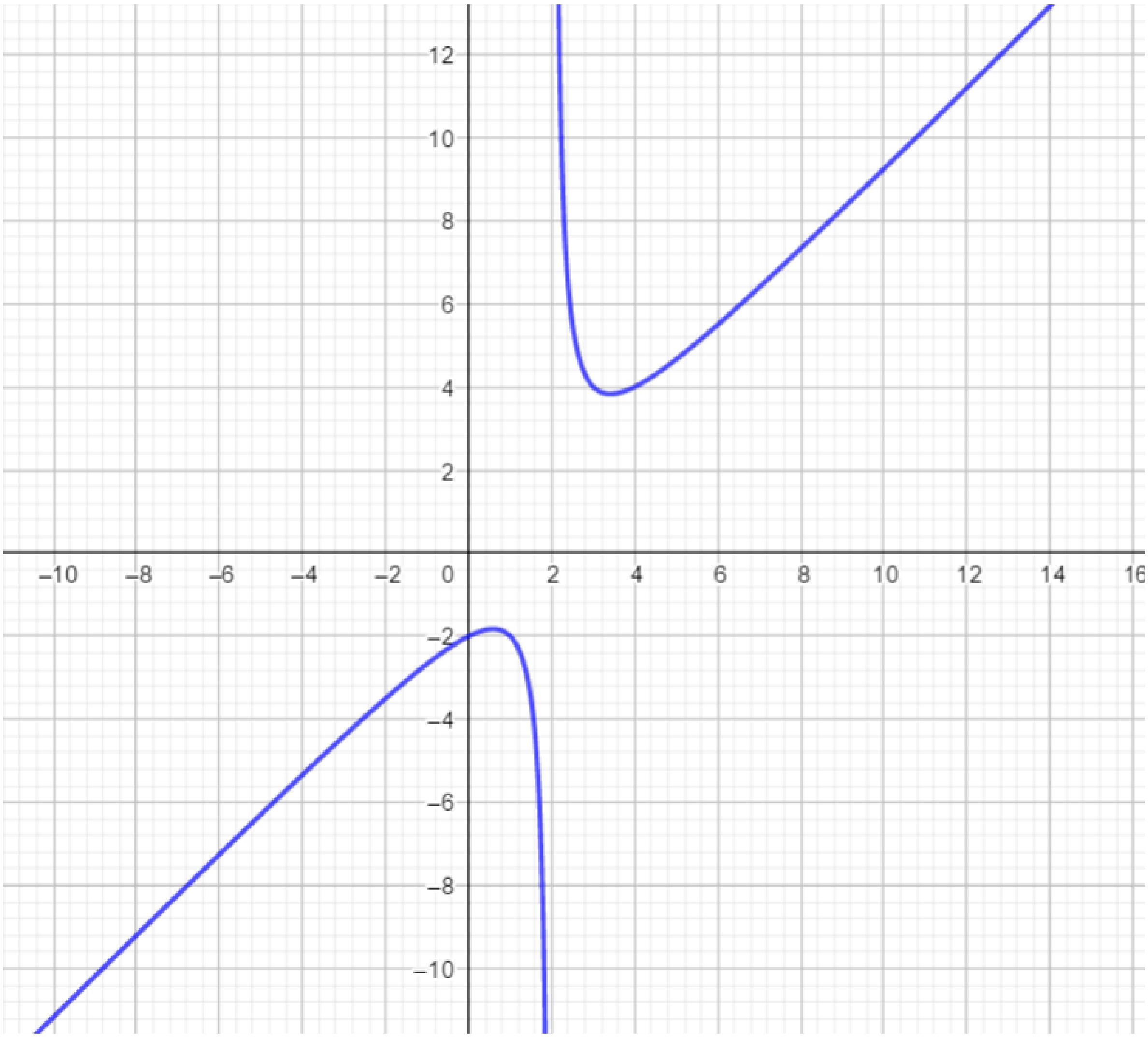
N(x) = 0 and Z(x) = 0 Gap at x

N(x) = 0 and N(x) \neq 0 pole at x.

For the above example, the following then applies:

N(2) = 0 and Z(2) = 6 ≠ 0 pole

The graph shows that no function values exist at the position x = 2. Similarly, the closer you approach 2 from the right or left, the more the function values tend towards infinity.



2.3. Points of intersection with the axes

2.3.1. Points of intersection with the x-axis

The points of intersection with the x-axis, the zero points, are determined by setting the function = 0. The equation $\frac{x}{y} = 0$ is true if the counter is 0. Thus, the zeros of a fractional rational function are determined by setting the zeros of the counter.

2.3.2 Intersection with the y axis

The point of intersection with the y - axis is obtained by calculating the function value at the position 0, i.e. determining f(0).

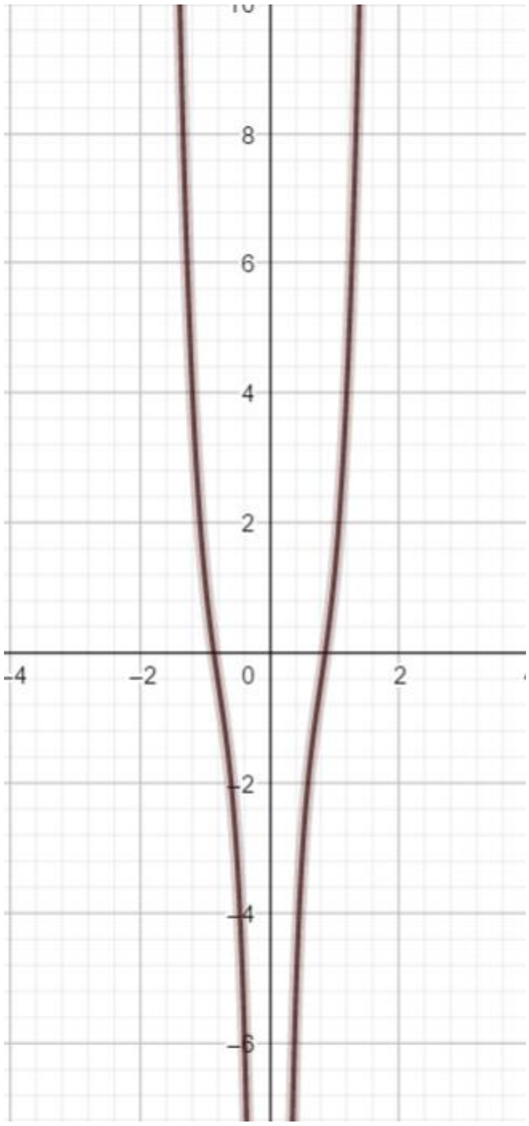
2.4. Symmetry

2.4.1. Axis symmetry

A function is axisymmetric to the y - axis if: f(x) = f(-x).

Example:

$$f(x) = \frac{7x^8 + 4x^2 - 5}{2x^4 + 3x^2}$$



$$f(-x) = \frac{7(-x)^8 + 4(-x)^2 - 5}{2(-x) + 3(-x)^2} = \frac{7x^8 + 4x^2 - 5}{2x + 3x^2} = f(x)$$

Thus the function is symmetrical to the y - axis.

2.4.2. Point symmetry

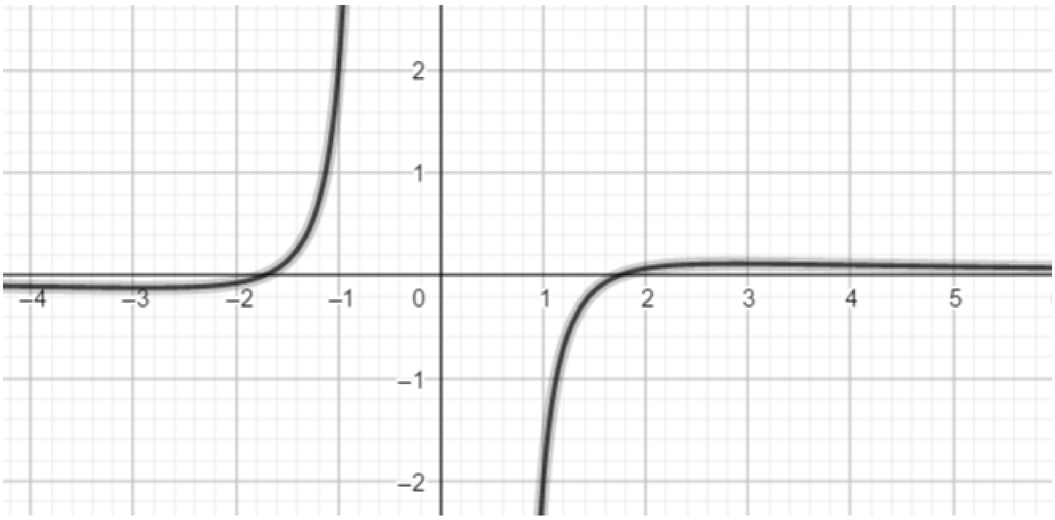
A function is point-symmetrical to the origin if: f(-x) = -f(x)

Example:

$$f(x) = \frac{x^2 - 3}{2x^3 - x}$$

$$f(-x) = \frac{(-x)^2 - 4}{2(-x]^3 - (-x)} = \frac{x^2 - 4}{-2x^3 + x} = \frac{x^2 - 4}{-(2x^3 - x)} =- \frac{x^2 - 4}{2x^3 - x} =- f(x)$$

The given function is thus point-symmetrical to the origin.



2.5 Extreme

Just as with completely rational functions, a high point is present when:

$f'(x_E) = 0$ und $f''(x_E) < 0$.

Similarly, a low point is present when: $f'(x_E) = 0$ and $f''(x_E) > 0$.

The first two derivatives are determined using the quotient rule.

Example:

$f(x) = \frac{x^3 - 9x^2 + 25x - 25}{x - 1}$

$u = x^3 - 9x^2 + 25x - 25 \rightarrow u' = 3x^2 - 18x + 25$

$v = x - 1 \rightarrow v' = 1$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{(3x^2 - 18x + 25) \cdot (x-1) - (x^3 - 9x^2 + 25x) \cdot 1}{(x-1)^2}$$

$$= \frac{3x^3 - 3x^2 - 18x^2 + 18x + 25x - 25 - x^3 + 9x^2 - 25x}{(x-1)^2} = \frac{2x^3 - 12x^2 + 18x}{(x-1)^2}$$

The zeros of this derivative are determined by setting the counter 0.

$2x^3 - 12x^2 + 18x = 0$

$x(2x^2 - 12x + 18) = 0 \rightarrow x_1 = 0$

In the brackets 0 must also come out. So you set this = 0 and solve the resulting quadratic equation with the help of the p - q - formula.

$2x^2 - 12x + 18 = 0 \mid : 2$

$\Leftrightarrow x^2 - 6x + 9 = 0 \mid p = -6; q = 9$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{(\frac{p}{2})^2 - p} = -\frac{-6}{2} \pm \sqrt{(\frac{-6}{2})^2 - 9} = 3 \pm \sqrt{0} = 3$$

So you have the two zeros $x_1 = 0$ and $x_2 = 3$.

These are inserted into the second derivative.

f‘(x) = $\frac{2x^3 - 12x^2 + 18x}{(x-1)^2}$

u = $2x^3 - 12x^2 + 18x$ → u‘ = $6x^2 - 24x + 18$

v = $(x - 1)^2$ → v‘ is determined using the chain rule:

z = x – 1 → z‘ = 1; g(z) = z^2 → g‘(z) = 2z

It follows with the chain rule for v‘: v‘ = g‘(z) · z‘ = 2 · (x – 1) · $1 = 2 \cdot (x - 1)$

f‘‘(x) = $\frac{u\prime \cdot v - u \cdot v\prime}{v^2} = \frac{(6x^2 - 24x + 18) \cdot (x-1)^2 - (2x^3 - 12x^2 + 18x) \cdot (2 \cdot (x-1))}{((x-1)^2)^2}$ | x – 1

kürzen

$= \frac{(6x^2 - 24x + 18) \cdot (x-1) - (2x^3 - 12x^2 + 18x) \cdot 2}{(x-1)^3} =$

$\frac{6x^3 - 6x^2 - 24x + 18x - 18 - 4x^3 + 24x^2 - 36x}{(x-1)^3}$

$= \frac{2x^3 - 6x^2 + 6x - 18}{(x-1)^3}$

Now insert the two zeros into the second derivative:

f‘‘(0) = $\frac{2 \cdot 0^3 - 6 \cdot 0^2 + 6 \cdot 0 - 18}{(0-1)^3}$ > 0 → Low point

f‘‘(3) = $\frac{2 \cdot 3^3 - 6 \cdot 3^2 + 6 \cdot 3 - 18}{(3-1)^3}$ = 0 →High point

The known method fails here, which is why we take a closer look at this passage. We therefore approach the point x = 3 from the left and right, by means of limit value observation. So we look at the gradient to the right and left of the point in question, which means that we calculate the limit value of the first derivative (gradient) at the point x = 3.

Approaching the 3 from the left means that the distance to the 3 becomes arbitrarily small, i.e.

3 – Δx. Δx becomes arbitrarily small, so it strives towards 0.

$f'(3 - \Delta x) = \frac{2(3-\Delta x)^3 - 12(3-\Delta x)^2 + 18(3-\Delta x)}{((3-\Delta x)-1)^2}$

$= \frac{6(\Delta x)^2 - 2(\Delta x)^3}{(2-\Delta x)^2} = \frac{(\Delta x)^2 \cdot (6 - 2(\Delta x))}{(2-\Delta x)^2} = \frac{\overset{>0}{\widetilde{y}} \cdot (\sim 6)}{(\sim 4)} = +$ → positive slopw

Now we approach the 3 from the right, thus inserting 3 + Δx into the first derivation:

$$\mathrm{f}'(3+\ \Delta\mathrm{x}) = \frac{2(3+\Delta x)^3 - 12(3+\Delta x)^2 + 18(3+\Delta x)}{((3+\Delta x)-1)^2}$$

$\frac{6(\Delta x)^2+2(\Delta x)^3}{(2+\Delta x)^2} = \frac{(\Delta x)^2\cdot(6+2(\Delta x))}{(2+\Delta x)^2} = \frac{\overbrace{y}^{>0}\cdot(\sim 6)}{(\sim 4)} = +$ → positive slope

Since the function increases before and after x = 3, there must be a saddle point at the point x = 3.

In order to determine the y - coordinates, the found zeros, $x_1 = 0$ and $x_2 = 3$, are inserted in f(x).

f(0) = …. = $\frac{-27}{-1} = 27$

f(3) = … = $\frac{0}{2} = 0$

The following coordinates are thus obtained:

Extremum (0 / 27); Saddle point (3 / 0)

on.