# Nummer 8

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1) a)

$22x - (44y - 18y + 5x - 3y - 4y + 77y - 72x)$

$22x - (44y - (-33y + 20x) - 72x)$

$22x - (44y + 33y - 20x - 72x)$

$22x - (53y - 52x)$

$22x - 53y + 52x$

$53x - 53y$

b) $3a^2 - 12ab + 4b^2 - (9a^2 + 12ab + 4b^2)$

$= 3a^2 - 12ab + 4b^2 - 3a^2 - 12ab - 4b^2$

$= 0$

c) $2ax + 4ay - ax + 6a \cdot 2b - 6y$

2) a) $\frac{3}{5} \cdot \left(\frac{2}{3} - \left(\frac{24+3}{12}\right)\right)$

$= \frac{3}{5} \cdot \left(\frac{2}{3} - \frac{27}{12}\right)$

$= \frac{3}{5} \cdot \left(\frac{8-81}{12}\right)$

$= \frac{3}{5} \cdot \left(-\frac{73}{12}\right)$ ; $\frac{1}{5} \left(-\frac{73}{-12}\right) = -\frac{73}{60}$

b) $\left(\left(\frac{65}{60} + \frac{52}{60}\right) \cdot \frac{4}{5}\right) + 9\frac{4}{6}$

$= \left(\frac{117}{60} \cdot \frac{4}{5}\right) + 9\frac{4}{6}$

$= \frac{117}{15} \cdot \frac{1}{5} + 9\frac{4}{6}$

$= \frac{117}{75} + \frac{4}{60}$

$= \frac{780}{500} + \frac{299}{500}$

$= \frac{1079}{500}$

c) $\left(\frac{2}{4} \cdot \frac{4}{3} - \left(\frac{1}{5} + \frac{3}{2}\right)\right) \cdot \frac{3}{8}$

$= \left(\frac{12}{12} - \left(\frac{2+6}{10}\right)\right) \cdot \frac{3}{8}$

$= \left(\frac{1}{3} - \frac{13}{10}\right) \cdot \frac{3}{8}$

$= \left(\frac{10}{30} - \frac{39}{30}\right) \cdot \frac{3}{8}$

$= -\frac{4}{30} \cdot \frac{3}{8} = -\frac{1}{3} \cdot \frac{1}{2} = -\frac{1}{6}$

3) a) $f(x) = \frac{\sin(x)}{\cos(x)}$

$u = \sin(x)$ $u' = \cos(x)$

$v = \cos(x)$ $v' = -\sin(x)$

$f'(x) = \frac{u'v - uv'}{v^2}$

$= \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot (-\sin(x))}{\cos^2(x)}$

$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

b) $f(x) = \frac{\cos(x)}{\sin(x)}$

$u = \cos(x)$ $u' = -\sin(x)$

$v = \sin(x)$ $v' = \cos(x)$

$f'(x) = \frac{-\sin(x) \cdot \sin(x) - \cos(x) \cdot \cos(x)}{\sin^2(x)}$

$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

c) $f(x) = \frac{\sin(x)}{2x}$

$u = \sin(x)$ $u' = \cos(x)$

$v = 2x$ $v' = 2$

$f'(x) = \frac{\cos(x) \cdot 2x - \sin(x) \cdot 2}{4x^2}$

d) $f(x) = \frac{x+2}{x-3}$

$u = x+2$ $u' = 1$

$v = x-3$ $v' = 1$

$f'(x) = \frac{1 \cdot (x-3) - (x+2) \cdot 1}{(x-3)^2}$

$= \frac{x-3-x-2}{(x-3)^2} = \frac{-5}{(x-3)^2}$

e) $f(x) = (x+2) \cdot \sqrt{x} = (x+2) \cdot x^{\frac{1}{2}}$

$u = x+2$ $u' = 1$

$v = x^{\frac{1}{2}}$ $v' = \frac{1}{2} x^{-\frac{1}{2}}$

$f'(x) = 1 \cdot x^{\frac{1}{2}} + (x+2) \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \sqrt{x} + \frac{1}{2} \cdot x \cdot x^{-\frac{1}{2}} + 2 \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \sqrt{x} + \frac{x^{\frac{1}{2}}}{2} + x^{-\frac{1}{2}}$

$= \sqrt{x} + \frac{\sqrt{x}}{2} + \frac{1}{\sqrt{x}}$

f) $f(x) = (\sin(x) + 2) \cdot x^3$

$u = \sin(x) + 2$ $u' = \cos(x)$

$v = x^3$ $v' = 3x^2$

$f'(x) = \cos(x) \cdot x^3 + (\sin(x) + 2) \cdot 3x^2$

g) $f(x) = (\sin(x) + 2x) \cdot (x-5)$

$u = \sin(x) + 2x$ $u' = \cos(x) + 2$

$v = x-5$ $v' = 1$

$f'(x) = (\cos(x) + 2) \cdot (x-5) + (\sin(x) + 2x) \cdot 1$

$= x\cos(x) - 5\cos(x) + 2x - 5 + \sin(x) + 2x$

4) a) $f(x) = \sin^2(x^2)$

$i = x^2$ $i' = 2x$

$a = \sin^2(i) = \sin(i) \cdot \sin(i)$

$u = \sin(i)$ $u' = \cos(i)$

$v = \sin(i)$ $v' = \cos(i)$

$a' = \cos(i) \cdot \sin(i) + \sin(i) \cdot \cos(i)$

$= 2\cos(i)\sin(i)$

$f'(x) = a' \cdot i'$

$= 2\cos(x^2) \cdot \sin(x^2) \cdot 2x$

$= 4x\cos(x^2) \cdot \sin(x^2)$

b) $f(x) = \cos(2x^3)$

$i = 2x^3$ $i' = 6x^2$

$a = \cos(i)$ $a' = -\sin(i)$

$f'(x) = -\sin(2x^3) \cdot 6x^2$

c) $f(x) = \sqrt{\sin(x)}$

$i = \sin(x)$ $i' = \cos(x)$

$a = \sqrt{i} = i^{\frac{1}{2}}$ $a' = \frac{1}{2} i^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2} \cdot \sin(x)^{-\frac{1}{2}} \cdot \cos(x)$

d) $f(x) = 2\cos(x^3 + 5)$

$i = x^3 + 5$ $i' = 3x^2 + 5$

$a = 2 \cdot \cos(i)$ $a' = -2\sin(i)$

$f'(x) = -2\sin(x^3 + 5) \cdot (3x^2 + 5)$

5) $f(x) = \frac{x^2 - 1}{x^3}$

a) $\mathbb{D}$: $x^3 = 0$ $x = 0$

$\mathbb{D} = \mathbb{R} \setminus \{0\}$

b) $S_x$: $y = 0$

$x^2 - 1 = 0$ | $+1$ | $\sqrt{\ }$

$x_{1/2} = \pm 1$

$S_{x_1}(-1|0)$ $S_{x_2}(1|0)$

$S_y$: $x = 0$

$f(0) = \frac{0^2 - 1}{0}$ ↯

$S_y$ ex. nicht

c) $(x^2 - 1) \cdot x^3 = 0 \mid :R$

$x_0 = 0$